Preliminary Orbit Determination by a Composite Multivariate Search Strategy

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A new strategy is presented for the problem of preliminary orbit determination with limited data and a very approximate initial estimate. This strategy, which is based on the minimization of the observation error function, is essentially a combination of two types of multivariate search, random and deterministic, in the parameter space, ordered in such a way as to exploit the relative inherent advantages of the two types of methods. The feasibility of achieving accurate state estimates with such a strategy has been explored by simulation studies. Further, the potential practical applicability of the proposed method has been demonstrated using a variety of actual tracking data sets and orbit conditions.

Introduction

SEVERAL methods have been reported in the literature[1,2] for the orbital state estimation of a satellite using noisy ground observation data such as range, range rate, azimuth and elevation angles, of which the differential correction method stands out as the most widely used. However, most of these approaches have an inherent disadvantage in that a close a priori state estimate is required to converge to the correct state rapidly. But such needs as construction of the orbital state quickly with the knowledge of only the prelaunch injection parameters, augmented by several minutes of single station tracking data, estimation of the state immediately after orbit correction maneuvers are encountered in practice. In order to handle such scenarios wherein the conventional methods may not always be able to accomplish the objective, several geometric methods have been devised which make use of certain combinations of observation types to give an approximate state estimate. Gauss's method, Gibb's method and the Double r-iteration are some of the widely used methods of this kind.[3,4] Though variations of Gibb's method have also been proposed in the literature that use specific observation types such as range alone, there is definitely a dearth for universal methods for preliminary orbit determination which can use any one or a combination of observation types. The optimization approach to minimize the weighted sum of the residuals is an obvious avenue that could be explored towards this end. In Ref. 5 there are reported the findings of a comparative study of applying several optimization techniques to the problem of orbit determination, using simulated data comprising more than one observation type and spread over two or three visible passes over a tracking station. In view of the existence of local minima, direct application of the deterministic search methods may lead to convergence to wrong states as can be seen in the results of Ref. 5. Such a problem, which makes preliminary orbit determination by direct search an unattractive approach, can be alleviated to a large extent by a suitable combination of both random and deterministic searches in the parameter space. In this paper such a strategy is proposed and its feasibility demonstrated by simulation exercises and real-world case studies.

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Method

The proposed composite strategy comprises both random and deterministic multivariate direct search in the parameter space[6] to minimize the sum of the residuals. The general philosophy underlying any multivariate search is to obtain better points in the search space successively by generating new directions and determining step sizes for advancement in these directions. In particular, random search methods[7,8] are that class of techniques wherein transition to successive candidate points involves pseudorandom number generators. In view of the randomness allowed during the course of the search, this class of methods in a sense "spread their wings" all over the search region and hence are capable of rapidly driving the search to the region of the optimum. On the other hand deterministic searches,[9] wherein transition between successive candidate search points are basically functions of the local characteristics, make a thorough search of a relatively small region around the point of initiation and hence are capable of exactly locating the optimum. In the present strategy, these inherent relative advantages of the two types of searches in the parameter space are exploited by suitably ordering the searches. The particular random search technique used in the present formulation is the Optimized Step Size Random Search (OSSRS).[10] In this method, the optimal step size for transition in a randomly chosen direction is determined by fitting a quadratic in terms of the step size. It can be seen that the behavior of this method is typical of random search methods in the sense that it is very efficient in the early stages of the search and staggers once the optimal region is attained. This is well illustrated by observing the convergence pattern for the following test function:

$$\min f(x) = (x_1 + 10x_2)^2 + 5(x_3 - x_4)^2 + (x_2 - 2x_3)^4 + (10x_1 - x_4)^4$$

The conventional starting point is (3, −1, 0, 1), at which the function value is 707336.0. The minimum is zero at (0,0,0,0). Table 1 shows the convergence pattern giving the number of function evaluations (NEE), the function value, and the parameter values at various stages of the search.

The deterministic search that is chosen is a variant, Improved Flexible Polyhedron Method (IFPM),[11] of the simplex method of Nelder and Mead[12] which is one of the simplest and most efficient direct search methods. The particular version chosen, established elsewhere[11] to be more efficient than several variants suggested by Parkinson and Hutchinson,[13] has been successfully applied to a number of problems drawn from a wide spectrum of fields.[14,15] As is conventional in the field of optimization, the yardstick for search strategy performance rating is the number function evaluation for convergence for certain test functions available in literture for the purpose. Table 2 gives the results using the original method, IFPM[11] and using the variant suggested in[13] for the following set of test functions.

Rosenbrock's parabolic valley function:

$$f(x)=100(x_2-x_1^2)^2+(1-x_1)^2$$

$$f_{\min}=0 \text{ at } (1,1), \text{ starting point: } (-1.2, 1.0)$$

Beale's function:

$$f(x)=\sum_{i=1}^{3}[C_i-x_1(1-x_2^i)]^2, \quad C_1=1.5; \quad C_2=2.25;$$

$$C_3=2.625 \quad f_{\min}=0 \text{ at } (3,0.5), \text{ starting point: } (0,0)$$

Box's exponential function:

$$f(x)=\sum_y \{\exp(-x_1y)-\exp(-x_2y) - x_3[\exp(-y)-\exp(-10y)]\}^2$$

where $y=0.1$ (0.1) 1.0 (i.e, y varies from 0.1 to 1.0 in steps of 0.10) $f_{\min}=0$ at (1, 10, 1), starting point: (0, 20, 20)

Rosenbrock's cubic function:

$$f(x)=100(x_2-x_1^3)+(1-x_1)^2$$

$$f_{\min}=10 \text{ at } (1,1), \text{ starting point: } (-1.2, 1.0).$$

Evidently, the improved version of the method scores over the original method and all the variants in Ref. 13.

The proposed strategy for Orbit Determination by Direct Search (ODDS), has these two search methods as its components. The cost function evaluation, being the calculation of the residues between predicted and given observations for the current value of the parameter vector, is common to both these methods. The switch from OSSRS to IFPM is made on the basis of a two-fold criterion for convergence that checks whether or not the same point in the search space continues to be the optimum for a specified number of iterations, or whether or not the decrease in the cost function is less than a preset positive quantity.

The orbit model that is used in ODDS is that of two-body including first-order (J_2 only) perturbations.

Simulation Study

Extensive simulation studies were carried out to explore the feasibility and limitations of the proposed strategy before its application to actual tracking data. The orbit selected for simulation was that of the satellite Rohini, to be launched by the Indian launch vehicle ASLV. The following nominal parameters were assumed:

$$a=6784.0387 \text{ km}$$

$$e=0.00218$$

$$i=45.5794 \text{ deg}$$

$$w=320.5947 \text{ deg}$$

$$\Omega=12.5058 \text{ deg}$$

$$M=39.2469 \text{ deg}$$

Epoch = 86-01-01-11-42-49-921

Adopting a two-body orbit model including first-order perturbations due to oblateness (J_2 only) of the Earth, tracking data comprising range, range rate and angles were simulated as observed from a station with coordinates 1 deg, S, 100 deg E and 0.0 km altitude. Bias and random (1σ) noise

Table 1 Typical convergence pattern of OSSRS

NFE	$f(x)$	x_1	x_2	x_3	x_4
1	707336.0	3.0	−1.0	0.0	1.0
5	55989.6	1.544	−0.733	−0.703	0.066
9	1034.5	0.458	−1.484	−0.185	−0.770
11	535.6	0.291	−1.007	−0.198	−1.633
13	23.3	−0.316	−0.312	−0.581	−1.634
28	9.6	−0.094	−0.098	−0.329	−1.607
88	0.3	−0.049	−0.002	−0.306	−0.493
226	$0.88*10^{-1}$	−0.039	−0.001	−0.264	−0.228
451	$0.77*10^{-2}$	0.0037	−0.0041	−0.118	−0.14
4006	$0.83*10^{-3}$	−0.0211	0.0019	−0.045	−0.045

Table 2 Comparison of results for test functions

	Original simplex (Kowalik and Osborne)		IFPM		Modified simplex (PHS)	
Function	NFE	Accuracy	NFE	Accuracy	NFE	Accuracy
Rosenbrock's parabolic valley function	200	6.6×10^{-8}	128	1.8×10^{-8}	135	5.8×10^{-8}
	–	–	153	1.4×10^{-11}	–	–
Beal's function	100	2.01×10^{-8}	63	2.07×10^{-8}	67	3.6×10^{-8}
			90	3.62×10^{-12}	–	–
Box's function	290	3.8×10^{-5}	210	7.84×10^{-9}	117	6.7×10^{-8}
Rosenbrock's cubic function	206	3.79×10^{-8}	142	8.64×10^{-9}	148	3.7×10^{-8}
	–	–	148	4.44×10^{-9}	–	–
	–	–	156	3.91×10^{-10}	–	–

Table 3 Effect of initial guess on convergence

Initial guess Number	Δr position error, km Initial	Final	ΔV velocity error in, m/s Initial	Final
1	172.5	0.4365	86.25	1.636
2	345.5	0.5705	173.29	4.23
3	518.75	1.0108	259.86	3.515

Table 4 Effect of data span on convergence

Data span, s	Δr position error, in km Initial	Final	ΔV velocity error, m/s Initial	Final
150	172.5	0.4365	86.25	1.636
60	172.5	1.1978	86.25	4.631
45	172.5	1.1534	86.25	6.359
30	172.5	0.683	86.25	80.7

Table 5 Effect of combination of observation types

Observation type	Position error, kms Initial	Final	Velocity error, m/s Initial	Final
$\rho, \dot{\rho}, Az, El$	172.5	0.4365	86.85	1.636
ρ, Az, El	172.5	0.1323	86.25	10.57
$\dot{\rho}, Az, El$	172.5	1.487	86.25	10.02
$\rho, \dot{\rho}$	172.5	0.523	86.25	4.436
$\dot{\rho}$	172.5	0.762	86.25	9.091
ρ	172.5	0.508	86.25	3.125
Az, El	172.5	1.66	86.25	26.8

in range, range rate and angles of levels 30 m, 0.3 m/s, and 0.1 deg, respectively were used for corrupting the observations. Several trend analyses were carried out using this set of simulated data. The first one, being aimed at establishing a convergence domain, was to study the effect of varying the initial guess. The accuracy of the converged elements can be quantified by Δr and ΔV which are the root sum of squares of the position and velocity component differences between actual and converged states. Table 3 gives the results for different initial guesses. The effect of data span on the accuracy of the converged state can be seen in Table 4 which indicates that curtailing the data span to less than 45 s introduces a sharp deterioration in the overall accuracy. This lower limit on the data span is, of course, a function of the level of observation noise introduced in the data. The next exercise was to assess the effect of various combinations of observation types on the final accuracy. From Table 5, which gives the results for all these cases, it can be inferred that all combinations except angles alone give satisfactory convergence with 150 s data span. Finally the effect of observation noise itself was studied by varying their levels in the data which is presented in Table 6.

Case Studies

Presented below are some of the results of applying the methodology and software of ODDS to actual tracking data of several satellites like INSAT-1A and BHASKARA-II. Wherever available, comparisons have been made with the results of orbit determination procedures which use elaborate orbit models and extensive observational data compiled during a week from more than one station. This helps to get an insight into the limitations on the accuracy of the present preliminary orbit determination scheme.

INSAT-1A

The tracking data of INSAT-1A, India's three-axis stabilized multiple payload communication satellite, fabricated and launched by the Ford Aerospace Corporation, comprises range, azimuth and elevation measurements.

Table 6 Effect of observation noise

Case no.	Ranger, m Bias	Random	Range rate m/s Bias	Random	Azimuth, deg Bias	Random	Elevation, deg Bias	Random	Δr, km	ΔV, m/s
1	100	100	1	1	0.3	0.3	0.3	0.3	2.8028	23.669
2	30	30	0.3	0.3	0.1	0.1	0.1	0.1	1.048	6.614
3	30	10	0.3	0.1	0.1	0.03	0.1	0.03	0.4365	1.636

Table 7 Results for INSAT Data

Orbit	Station	Data span		Orbital elements a	e	i	w	Ω	M
Transfer orbit	Andover 289.3 deg E 44.634 N 19 m	8 min	Initial guess	24006.9	0.726	28.001	179.001	108.237	142.77
			Odds result	24028.66	0.725	28.064	178.873	108.471	138.78
			Assumed (Ford Aerospace)	24035.34	0.723	27.998	178.99	108.23	139.35
Synchronous orbit	Hasan 76.098 deg E 13.07 deg N 901.6 m	5 min	Initial guess	42003.167	0.004	0.044	302.13	337.85	250.53
			Odds result	42160.479	0.0001	0.049	327.401	331.699	231.256
			Assumed (Ford Aerospace)	42163.915	0.0004	0.056	327.294	331.588	231.475

Accuracies of 50 m in range and 0.05 deg in angles have been quoted (private communications with INSAT space segment office, Bangalore) for these observation types. Two tracking data sets, one during transfer orbit and the other during synchronous orbit, were used to test the algorithm. Table 7 gives the station coordinates, the initial guess, and the converged elements in each of the cases. The initial guess used in the transfer orbit case was obtained using the first and the last observations as follows

Let ρ_1, Az_1, El_1 and ρ_2, Az_2, El_2 be, respectively, the first and the last observations. Transformation 7 of Ref. 4 from position in the azimuth-elevation coordinate system to position in the right-ascension-declination coordinate system would give the position vectors $\vec{r}_1$ and $\vec{r}_2$. The state vector at time $t_0 = (t_1 + t_2)/2$ can be calcualted using

$$\vec{r}_0 = (\tau_2/(A\tau_2 - B\tau_1))\vec{r}_1 - (\tau_1/(A\tau_2 - B\tau_1))\vec{r}_2$$

$$\dot{\vec{r}}_0 = (A/(A\tau_2 - B\tau_1))\vec{r}_2 - (B/(A\tau_2 - B\tau_1))\vec{r}_1$$

where

$$\tau_1 = (t_1 - t_0),\ \tau_2 = (t_2 - t_0),\ A = (1 - \tau_1^2/2r_0^3)$$

$$B = (1 - \mu\tau_2^2/2r_0^3) \text{ and } r_0 = (r_1 + r_2)/2$$

The state vector $(\vec{r}_0, \dot{\vec{r}}_0)$ after a transformation gives the orbital elements which have been used as the initial guess. The set of orbital elements for comparison in this case was that obtained from orbit determination program supplied by Ford Aerospace using two stations data spread over 36 h. Figures 1a-c show the observation residuals throughout the duration of the data span for the transfer orbit case.

Bhaskara-II

Bhaskara-II, India's second satellite for Earth observation, was launched in November 1981 at a Soviet Cosmodrome. Single-pass range, range rate, and direction cosines data of this satellite over a single station were taken up individually to test ODDS.

Direction-Cosines Data

By direction cosines ℓ, m are meant the cosine of the angles between the station-spacecraft vector and the axes pointing towards the east and the north in the local tangent system, i.e., $\ell = y_h/\rho$ and $m = x_h/\rho$, where (x_h, y_h, z_h) are the components in the topocentric coordinate system and ρ is the range.

Table 8 Converged elements for various initial guesses

Case no.	a	e	i	ω	Ω	M
1	7100.0	0.0	49.75	137.747	174.208	95.293
	6915.5	0.001	50.694	133.36	183.633	326.532
2	7200.00	0.0	49.75	137.747	174.208	95.293
	6905.9	0.00248	50.55	138.752	184.094	320.998
3	7400.00	0.0	49.75	137.747	174.208	95.293
	6905.50	0.0015	50.58	128.258	184.189	331.291
4	7500.0	0.0	49.75	137.747	174.208	95.293
	6900.00	0.0015	50.67	128.254	183.735	331.626
5	7600.0	0.0	49.75	137.747	174.208	95.293
	6902.4	0.0017	50.52	127.941	184.193	331.663
6	7700.0	0.0	49.75	137.747	174.208	95.293
	6900.0	0.00163	50.62	127.757	183.888	332.020
7	7700.0	0.0	48.75	137.747	174.208	95.293
	6902.6	0.00182	50.74	129.721	183.499	330.336
8	7050.0	0.0	47.75	137.747	174.208	95.293
	6897.3	0.00153	50.61	139.805	185.931	319.963
9	7050.0	0.0	46.75	137.747	174.208	95.293
	6903.4	0.0015	50.619	138.327	183.893	321.464
10	7050.0	0.0	45.75	137.747	174.208	95.293
	6903.4	0.0019	50.56	129.903	184.087	329.781
OD result	6902.34	0.002006	50.637	131.466	183.601	328.314

NOTE: First line is the initial guess and the second line is the converged solution in each case. Station coordinates: 11.08°E, 47.81°N, 0.0 km; data span 4 min 30 s.

Table 9 Results for range and range rate data

		a	e	i	$w+M$	Ω
Data type, time span	Initial	6964.273	0.02205	49.942	37.542	115.774
Range rate	Converged	6920.499	0.0055	50.476	46.943	116.128
7 min	Actual	6909.481	0.0024	50.603	46.593	116.049
		Position, kms	Velocity m/s			
Position and velocity errors	Initial	170.0	97.52			
	Final	27.6	25.94			
Range data	Initial	7245.0	0.042	50.374	14.599	332.636
2 min	Converged	6907.356	0.0025	50.599	22.607	332.292
	Actual	6902.867	0.0021	50.642	22.684	332.313
		Position, kms	Velocity m/s			
Position and velocity errors	Initial	173.2	86.0			
	Final	0.378	9.1			

NOTE: Station coordinates: 13.67°N, 80.19°E, 0.078 km.

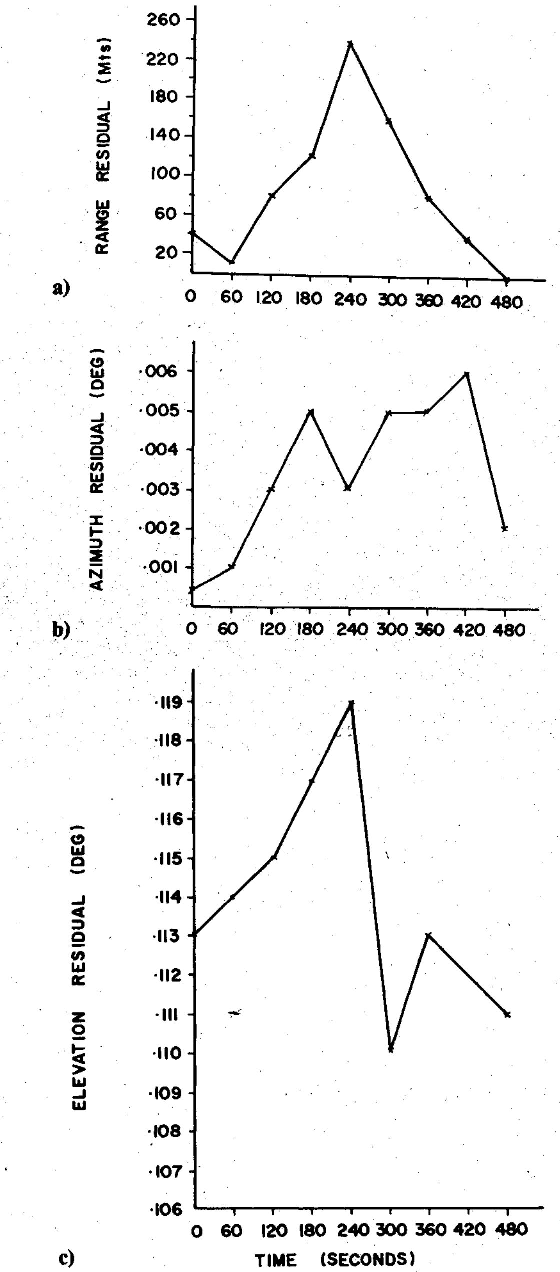

Fig. 1: a) Range residual vs time; b) azimuth residual vs time, c) elevation residual vs time.

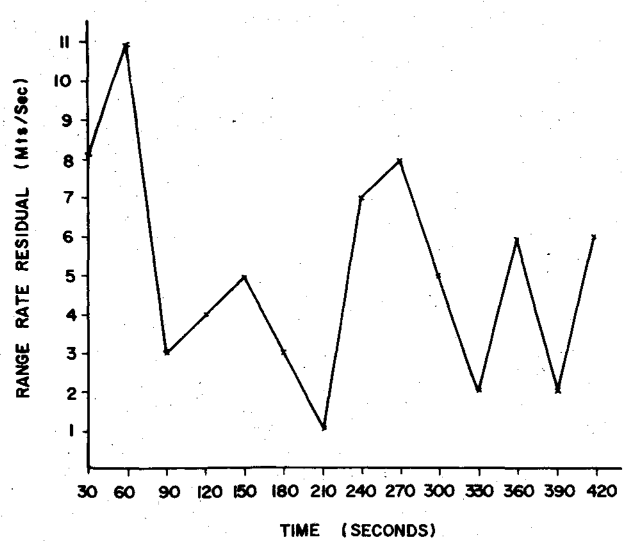

Fig. 2 Range rate residual vs time.

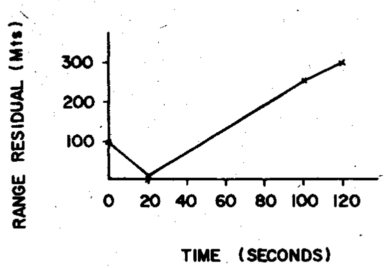

Fig. 3 Range residual vs time.

The initial guesses used to initiate ODDS and the converged elements obtained using each of these are given in Table 8, along with those obtained by an elaborate orbit determination. The latter was obtained from data over seven days using an orbit determination software Satellite Orbit Improvement Program (SOIP)[16] which is based on R.H. Merson's analytical theory for satellite motion and is essentially a linear, iterative differential correction method. The orbit model considered therein includes the effects of asphericity of the Earth and the atmospheric drag, and is applicable to close-Earth satellites. It is seen in the table that the solutions obtained using ODDS are close enough, even when the initial guesses are very inaccurate.

Range and Range Rate Data

Range and range rate data were individually used as measurements for ODDS. Table 9 gives the details of the data sets and the accuracies of the converged elements obtained using them. Figures 2 and 3 show the residual error between the observed and predicted measurements throughout their respective data spans.

Conclusions

The concept of a composite strategy comprising both random and deterministic search methods is presented for the problem of preliminary orbit determination using single-pass tracking data and a very approximate initial guess. Invoking a random search at the start alleviates the problem of local minima to a great extent and introduces rapid convergence to the correct orbital state. The simulation studies presented serve to show the feasibility of such a methodology, whereas the application to several real-world case studies demonstrates the capability of the proposed strategy as a potential tool for rapid orbit determination. The effective use of the proposed methodology for a rapid assessment of orbit maneuver system performance can be envisaged. Also, in view of the simplified nature of the process, such an approach would be relevant to onboard autonomous satellite orbit determination.

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